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Applications of Fibonacci Sequences and Golden Ratio

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Abstract

The study mainly focuses on the use of the Golden Ratio and the Fibonacci sequence. The connection between them can be clearly visible in nature. With the help of the Fibonacci sequence scientists have solved many mysteries related to nature. Everything that is around us somehow or other depends on Fibonacci numbers, the Golden Ratio, and the Fibonacci sequence. Some examples are –Flower petals- Lily, Rose, Daisy, Marigold, Sunflower, Iris, Buttercups, wild rose, larkspur Trillium, Bloodroot, Aster, and Susan; Seed heads-Sunflower; Snail; Fruit-Apple, Banana, Pineapple; Human Face; Tree Branches; Cyclone; Pinecones; Shells; Spiral Galaxies; Bees; Famous architecture design – Taj Mahal, in Hindu rituals, in decoding-coding the data, in providing security to the sensitive data and all over the world, in mother's womb (about her baby's position), etc. The current study reflects that there is no limitation to the Fibonacci pattern and Golden Ratio in our surroundings.

Keywords

Fibonacci Sequence, Golden Ratio, Fibonacci number, Hindu rituals, baby womb.

1. Introduction

Mathematics is a branch of science which deals with numbers, shapes, order, relations, sequences, series. It quickens our life and makes it easier to understand various real-life applications. Real Analysis is a branch of mathematics that makes us aware of the nature of real numbers, sequences, series of real numbers and real function. Sequences are useful when we try to do approximate calculation, run rate, estimate score. In Real Analysis, sequences play a vital role in understanding the nature of real numbers, their convergence and divergence nature. In sequence the numbers are arranged in particular manner and follow the same pattern. There are 4 main branches of sequence:

Arithmetic Sequence: It contains numbers in a manner that if we subtract the two consecutive digits then the answer remains same. For example: 3, 5, 7, 9, 11, 13, 15….in this sequence we observe that we have common difference of "2" between two consecutive numbers.

Geometric Sequence: It contains a number in a manner that if we divide two consecutive digits then the answer remains constant. For example: 5, 10, 20, 40, 80, 160, 320, 640…..in this sequence we observe that we have common ratio of "2" be-tween the consecutive digits.

Harmonic Sequence: It is a sequence formed by the reciprocal of an arithmetic progression. For example: 1/3, 1/5, 1/7, 1/9, 1/11….in this sequence we observe that if we take the reciprocal of each number then the arithmetic sequence is generated.

Fibonacci Sequence: It is a sequence formed by the addition of two numbers to obtain the next number of the sequence. There is no visible pattern. For example: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144……….

0+1=1

1+1=2

2. Fibonacci Sequence

A great Italian Mathematician Leonardo Pisano was the 1st person who brings the concept of Fibonacci sequence and number in existence. His nickname is "Fibonacci ", and he was popular by his nickname. A Fibonacci Sequence is the one in which every number obtained is an addition of the previous 2 numbers. The Fibonacci Numbers can be defined by the relation.

$$f_n = f_{n-1} + f_{n-2}$$

For all n>=3, where Fn represents the nth Fibonacci Number (here n is called an index).

The Fibonacci Sequence

1,1,2,3,5,8,13,21,34,55,89,144,233,377…

1+1=2	13+21=34
1+2=3	21+34=55
2+3=5	34+55=89
3+5=8	55+89=144
5+8=13	89+144=233
8+13=21	144+233=377

Figure 1. The Fibonacci Sequence

3. History

An experiment performed by Fibonacci to understand the beauty of Fibonacci numbers and their Sequence is with the growth of the rabbit population. He thought that if 2 newborn rabbits are put in a pen, then "How many rabbits will be in the pen after a year?" To know about the number of rabbits in a pen after a year, he takes 2 rabbits and puts them into a pen with the assumption that:

- Rabbits always give birth to a male and female offspring
- Rabbits can reproduce once every month.
- Young Rabbits can become a pair of adult rabbits in a month.
- Rabbits in pen never die.

- Rabbits are healthy and have no disease or problems

Then he starts observing them, after a month no adult pair was there but one offspring was there, in a second month one adult pair and zero offspring was there, in third month one adult pair and one offspring, in fourth month two adult pair and one offspring and this continuo.

At last, there were 89 adult pairs and 55 offspring. At the start of 12th month or last month i.e December- Total pairs of rabbits are 144 and total number of rabbits are 288. But at last, we have Total 233 pair and 466 rabbits in a pen. This rabbit population experiment is also discussed in research paper 9 (present in reference section). Here is figure 1 which explains how the Rabbit population increases every month. From 1 to 2, 2 to 3, 3 to 5, 5 to 8 and so on.

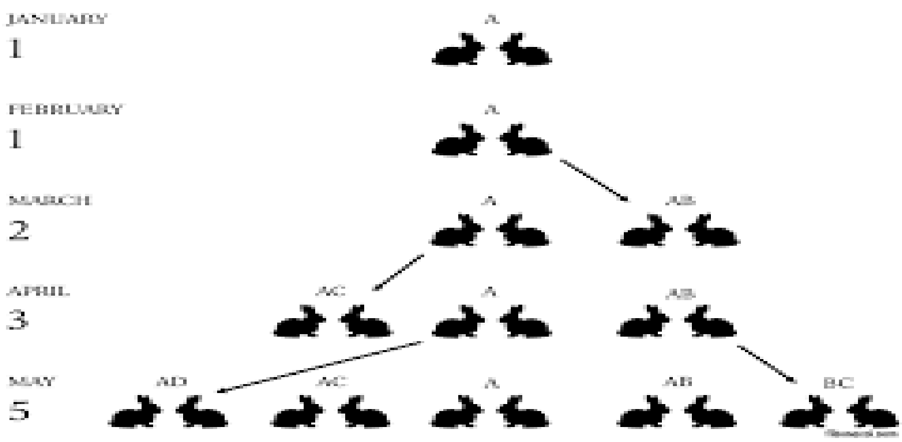

Figure 2. Rabbit Population Growth

After this experiment, one question arises in my mind - "Is the rabbit population is the only real-life application of Fibonacci Sequences." When I future investigated and went through the many research journals and articles, I learned about the Golden Ratio and its relationship with this sequence. I also found many real-life applications of Fibonacci numbers, its sequence, and Golden ratio.

4. Golden Ratio

The Golden Ratio was introduced in 1800's. Martin Ohm was the 1st human who uses the term "golden" to describe the Golden Ratio. It is a special mathematical relation. The two numbers are in Golden ratio if the addition of number (c, d) is divided by the greater number among the two (c) is in similar proportion to the greater number when divided by smaller number (c/d). Mathematically it is written as,

$$(c+d)/c=c/d$$ where c is the larger side and d is smaller side.

The value of Golden Ratio is approx. 1.618. It is denoted by the letter phi. The symbol for Golden Ratio is shown in the figure below:

Figure 3. The Golden Ratio

5. Link between Fibonacci Number and Golden Ratio

A Golden Ratio is best estimated by the prominent "Fibonacci Numbers." Fibonacci Numbers are a continuous sequence beginning with 0 and 1 and the next number is calculated by adding the preceding two numbers. The Fibonacci sequence is 1, 2, 3 ,5, 8, 13 and so on.

1(0 1)

2(1 1)

3(2 1)

5(3 2)

8(5 3)

And so on……

Now, if we calculate a ratio of 2 two consecutive numbers then we observe that the greater the Fibonacci numbers, the closer it will be to the Golden Ratio, i.e., 1.618.

2/1=2

3/2=1.5

5/3=1.6666666666

8/5=1.6

13/8=1.625

21/13=1.615

And so on…….

The Golden Ratio is occasionally called as the "Divine proportion", because of its application in the natural world.

6. Importance of 1.6184

This value 1.6184 is nothing but the Golden Ratio. Most of the things around us are somehow related to this number. These numbers themselves are sufficient to solve the great mystery of nature. Even the ratio of the two consecutive Fibonacci numbers is approximately equal to this value i.e., 1.6184. The most surprising fact about this value is that if we multiple 0.618 by any Fibonacci number then the result obtain is also the Fibonacci Number. As we see in the above figure, if we multiple 5 with 0.618 the number we obtain is approximately 3, which is the Fibonacci number. In a similar manner if we multiply 8 with 0.618 then the number obtained is approx. 5, which is again the Fibonacci number.

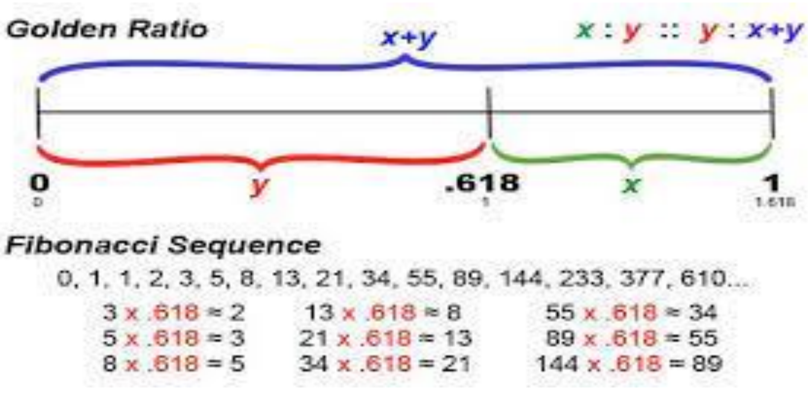

Figure 4. Relation of Sequence with Ratio

7. Fibonacci and Golden Ratio in Plants

Flower petals: Have you ever thought that most of the flower petals are three, five, eight, thirteen or twenty-one in number. Have you ever closely examined flower petals? If yes, then you will observe some pattern or sequence in it. And if you more closely try to examine it you will be surprised to know that they follow “Fibonacci pattern” and are arrange in “Golden Ratio”. The petals of the flower follow the Fibonacci sequence. Some of the examples are the Lilies and Iris, which has three petals, Buttercups, wild rose, larkspur has five petals, Daisy has 34 petals, Marigold has 13 petals, Trillium has three petals, Bloodroot has 8 petals, Aster and Susan have 21 petals, Plantain have 34 petals Delphiniums have 8 petals.

The golden ratio is also applicable in petals as the arrangement of petals is such that they are placed at 0.618 per turn. Nature also favors this angle. The sunlight and other important factors required for its growth are perfect at this angle. Sunflower is a big, beautiful flower showing the Fibonacci pattern, in it the spirals are in the center of the flower following the Fibonacci Sequence 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, and so on. In this flower, there are a series of arcs that are in opposite directions. In total, there are 21 petals which are a Fibonacci number.

Figure 5. Flowers having Fibonacci number petals

Seed Heads: The Fibonacci process is also applicable in the head of the flowers. As is known by us that the seeds are produced from the centre of the flower and then it gradually shifts towards the outer side to fill all the space. The seed head following the Fibonacci pattern can be easily seen in the sunflower. In some of the flowers the seeds heads are so closely arranged that they cannot be counted. In sunflower there are two series of curves that are in different directions. In it, the seeds are arranged at a golden angle from each other and create a beautiful spiral. These spirals are packed so tightly that it is difficult to count the seed heads. The Fibonacci pattern allows the flower to bend a large number of seeds in their heads. As the seed grows, the head shifts the seeds towards the side to make space for new seeds. Thus follow a Fibonacci pattern.

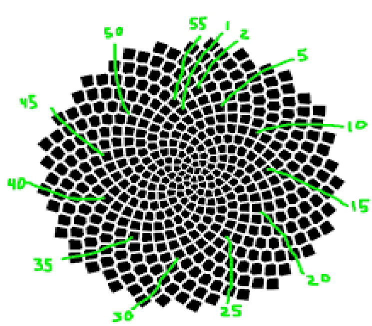

Figure 6. Seed Heads

Pinecones: The seed pods on a pinecone are arranged in a spiral pattern. Each cone consists of a pair of spiral patterns. The number of the spiral found on pinecone is surprisingly a Fibonacci numbers. If we closely observe the pinecones, then we observe that a cone has eight spirals on one side and thirteen spirals on the other side. We know that both eight and thirteen are nothing but Fibonacci Numbers. Similarly, if we observe a larger pinecone, we can easily observe it has twenty-one spirals starting from base to end, which is again a Fibonacci number. Here below is figure 7 which shows how the spirals followed the Fibonacci Pattern.

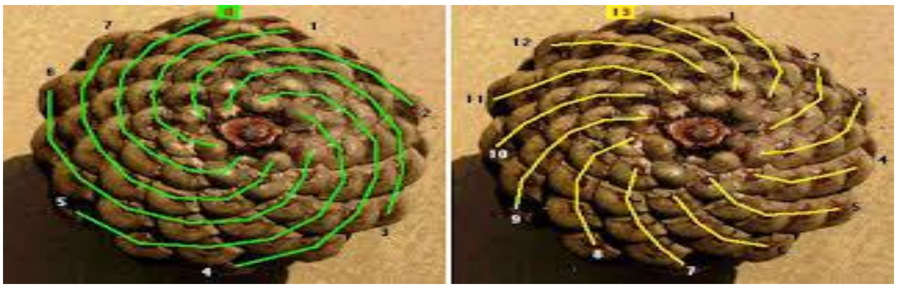

Figure 7. Pinecones

8. Fibonacci and Golden Ratio in Animals

The Fibonacci Sequences are not only limited to numbers, but they also help us to describe the shape of certain elements or things or creatures that are present in nature. Some of the shapes that Fibonacci sequences describe are- Logarithmic Spirals and Nautilus shells. The spiral pattern is seen in snail shells, in the sleeping positions of the many animals-cats, dogs, in certain spider webs, and in the horns of many cows, goats, sheep, and starfish. The sheep horns form a spiral shape. Not only in sheep's horns but also in the horns of goats, cows, deer such a pattern of the golden spiral is seen. A similar pattern is seen when many domestic animals sit in a sleeping position, some of them are cows, dogs, and cats.

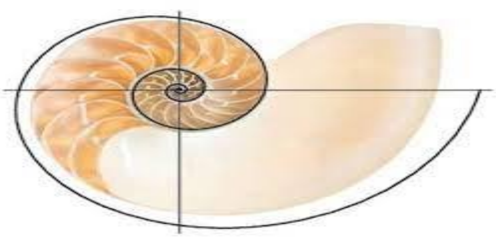

Figure 8. Spiral pattern in Snail

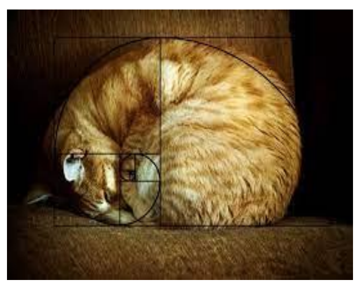

Figure 9. Cat sleeping position

9. Golden Ratio in Spiral Galaxies and Cyclones

The Fibonacci sequence and Golden ratio is not only limited to our surroundings, but it is also present in our solar system. The milky way galaxy has many spiral arms that follow the Fibonacci pattern. If we look at the figure of cyclone then we observe that the cyclone also originated in the spiral form, and somehow it is related to Fibonacci pattern and golden ratio.

(a) (b)

Figure 10. Fibonacci and Golden ratio in (a) sprial galaxi and (b) in Cyclone

10. Golden Ratio in Spiral Galaxies and Cyclones

The golden ratio and Fibonacci sequence are also useful in art, architecture and in graphical design. With this pattern the elements are placed in such a manner that it attracts the viewer and creates a visual proportion in a design. If we have a look on our historical monuments, we will see that many of them are designed on the Fibonacci pattern, and they form golden spiral shape. The best example of historical monument has Fibonacci pattern and golden ratio is- Taj Mahal. Every one of us is familiar with the most mysterious painting. The painting of Mona Lisa, made by Leonardo da Vinci. The paintings of Mona Lisa also follow the Fibonacci pattern and form the golden spiral. In this painting we can easily draw the golden rectangle and can see the golden ratio. Modern artists and architecture believe that the Fibonacci pattern fascinates the people more than the any other pattern. The evidence that was found shows that not only modern architecture, but also ancient architecture believed that golden spiral pattern is more fascinating than any other pattern.

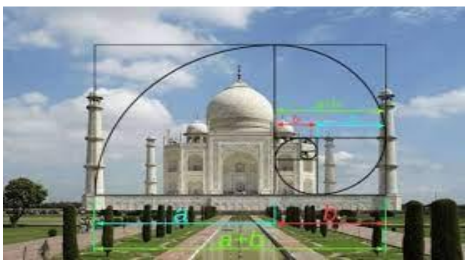

Figure 11. Golden ratio in Taj

Figure 12. Golden ratio in Monument

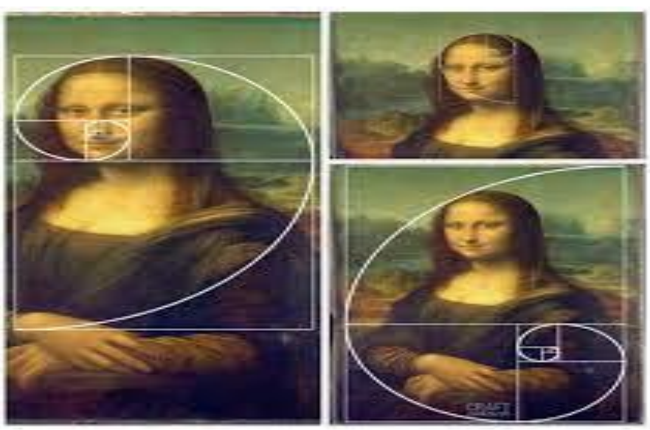

Figure 12. Fibonacci and Golden Ratio in monalisa

Figure 13. Fibonacci and Golden Ratio in designing

11. Fibonacci and Golden Ratio in Human Body

The human body, created by God also followed the golden ratio. If we stand in front of the mirror and see: Starting with the face-from top to bottom, it follows golden ratio. The distance between the nose and mouth are each at golden sections with the distance between the eyes and the bottom of the chin. Similar proportion is also be seen from both the sides and the eyes and ears. Now if we look at the fingers in each hand we see that they are five in number, toes are 2 , hands 2, each finger divide into 3 parts, thumb 2, if we try to conclude we see that each number is Fibonacci number.

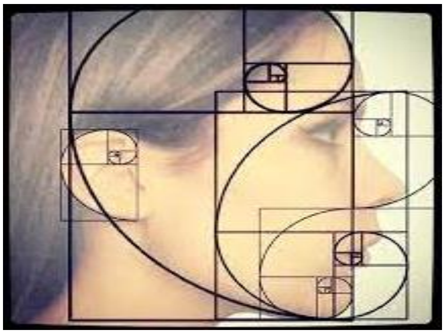

Figure 14. Fibonacci in face

Figure 15. Fibonacci in hand

12. Fibonacci in Reproductive Process of Bees

If we consider a colony of bees, then we observe that the ratio of number of females in the colony to the number of males in the colony is equal to the approximate to 1.618, which is nothing but a golden ratio. Similarly, the family tree of bees also follows the same pattern. The male has one female bee as a parent whereas females have one male and female bee as a parent. When it comes to family tree, male bees have 1,2, 3, 5, 8 as parent, grandparent and gr grandparent respectively. And the same pattern, female bees have 2, 3, 5, 8, and so on as parent, grandparent respectively.

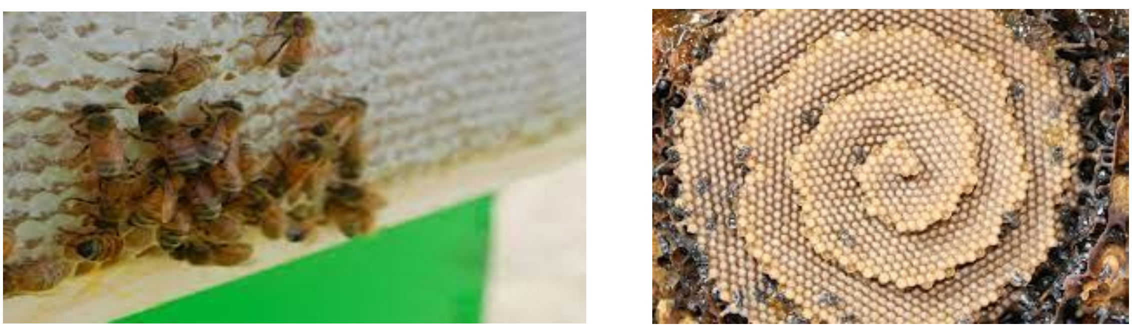

Figure 16. Golden Ratio in bees

13. Fibonacci in Fruits and Vegetables

If we cut a fruit into two parts and observe the fruits like apples, pear, lemon we will see that they all are having the seed in Fibonacci Numbers – 3, 5, 8, 13 and so on. Also, in some fruits like lemon and orange, if we cut it into a slice we see that the spiral shape is present in it. This is how the fruits have Fibonacci pattern and golden spiral. Similarly, if we cut a vegetable like gourd into two pieces and see then we come to know that the seeds are present in the Fibonacci Numbers like 3, 5, 8, 13, 21 and so on. This is how the vegetables follow the Fibonacci number and golden ratio.

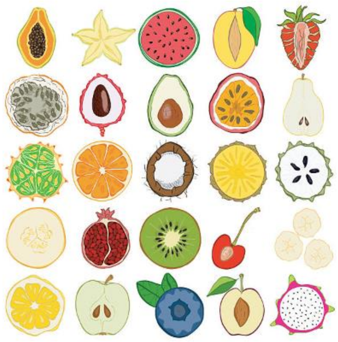

Figure 17. Fibonacci in Fruits & Vegetable

14. Fibonacci in Coding and Decoding

The Fibonacci Sequences and Golden Ratio is very helpful in protecting the data and in providing security to the data or information. The Fibonacci number and golden ratio can be used to code or decode the data or information. Coding is the process of writing the information or data in the coded form or cipher text, whereas decoding is the process of converting the coded data or cipher text into a simple text.

For example: Let us suppose that there is person named Roy wants to send a message to his friend Arav that the "Fibonacci is unique sequence". But the channel from where the message passes is not secure so with the aid of Fibonacci numbers, he coded the original message and sent it to Arav then Arav will decode the message with the aid of key.

Firstly, original message or information or data is- "FIBONACCI IS UNIQUE SEQUENCE".

Key: A

If Roy takes "A" as a key. And then he wants to code data using Fibonacci sequence so the code for original message will be-

Plain text: F I B O N A A C I I S U N I Q U E S E Q U E N C E

Cipher text: A B C C H M U H C Q L E M L Y K J I S U A C E O Q

This coded data is only decoded by the person who has key and a knowledge of Fibonacci numbers and their sequence. Here Arav has both key and knowledge about Fibonacci numbers. Now when Arav receives this message it will be in coded form so he will use a key and decode the code into a original message.

Cipher text: A B C C H M U H C Q L E M L Y K J I S U A C E O Q

Plain text: F I B O N A C C I I S U N I Q U E S E Q UE N C E.

As we see in the above example, Roy coded the original message with the aid of Fibonacci number and a key and sent the message to Arav. Then again with the aid of Fibonacci numbers and a key Arav decoded the message. This is how Fibonacci numbers and golden ratios are useful in securing and protecting the data or information or message.

15. Golden Ratio in Security

With the development of the modern technologies the security and protection for data, information, or messages from the attackers become a challenge. This challenge can be overcome by the golden ratio concept. As we all are aware that the value of the gold ratio is approximately 1.6184. SO, we can use this value to lock the files or sensitive data. For example: Let us suppose that Roy has some sensitive file that he wants to send to Riya. But he was afraid of the attacker or hacker that may damage or misuse the sensitive data of the file. Now he was in the dilemma of how to send it to Riya. Solution: Roy can create a program in which he can secure his file by the pattern. Now, let us see how Roy creates a program with the pattern in such a manner that when Riya enters the key (which only Riya and Roy know), the program asked to enter another number such that the division of the two numbers is 1.6184 i.e., a golden ratio. The person who has no knowledge about the golden ratio and the numbers which on dividing give 1.6184 as a result. Even if the attacker comes to know that 1.6184 is a value of golden ratio, then also the attacker will not be able to hack the file since there are infinite number in Fibonacci sequence.

16. Golden Ratio in Security

If we closely examine the practice and rituals of different religion, we will notice that each ritual in connected to Fibonacci number in a similar pattern. For example, A Navratri is of 9 day which is addition of one to a Fibonacci number 8. In total if we see then Hindus have 4 Vedas which is obtained by the addition 1 to a Fibonacci number 3. Our God Shree Ram got 14 years of inheritance, the number 14 we can obtained by adding 1 to the Fibonacci number 13. Navratri occurs twice in a year, here the two is itself a Fibonacci number and also, we can obtain two by adding 1 to the previous Fibonacci number 1. It is believed that there are 3 Avatars of the supreme power- Brahma, Vishnu, Mahesh.

17. Fibonacci Number and the World

There are many mysterious places and things in the world which we come across. We all see many things or study about many places or never observe the pattern that they all follow. Let us see how they follow and what they follow. There are 12 Jyotirlinga sites, here a number 12 can be obtained on subtracting one from a Fibonacci Number 13. There are 7 wonders in the world, seven sisters in India, 7 continents in the world, 7 oceans of the world here the number seven can be obtained by subtracting 1 from the Fibonacci number 8.

18. Fibonacci and Golden Ratio in growth of Baby in Womb

We all are aware of the fact that the Fibonacci number is applicable in our whole body. This made me think of the time period when the baby was in mother's womb. If we try to examine Figure 5.1, we will see that the baby is resting in mother's wombs in golden spiral shape, which is formed with the aid of golden rectangle. This shows that it follows the golden ratio. Even if we try to enhance our knowledge about the gestation period it is 7 to 9 months. These two numbers can be easily obtained by subtracting one from 8 and adding one to 8 respectively. This shows that the child in the mother's womb is somehow interconnected to Fibonacci Numbers.

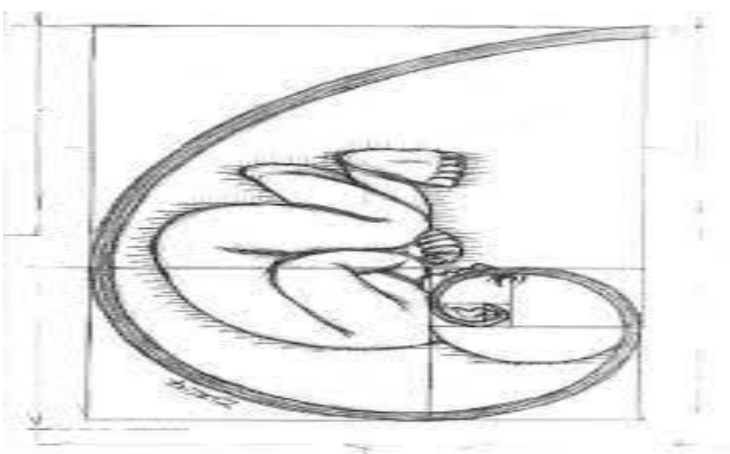

Figure 18. Fibonacci Sequence and Golden Ratio in unborn child

19. Conclusion

The Fibonacci numbers are nature friendly numbers. They have the power to define everything around us in some way. An interconnection between the Fibonacci Numbers and sequence with Golden Ratio gives us a lot of advantages from providing security and protection to sensitive data to code and decode the data. Fibonacci Numbers are present everywhere in petals arrangements, seeds head, in pinecones, in fruits and vegetables, in architecture and art. Today there is no field that remains untouched by Fibonacci numbers and Golden Ratio. Everything around us is connected to one another in some ways. If we try to examine the natural things around us, we will surprisingly come to know that those things follow Fibonacci Sequences and form a golden spiral. I chose this topic so that I can see the beauty of Fibonacci Numbers and its real application around us. This topic not only enhances my knowledge but also makes me aware of nature friendly numbers. Let us finish by the words- “START OBSERVING THINGS AROUND YOU, LET SEE HOW GOD USE MATHEMATICS IN DESIGNING A UNIVERSE”.

References

[1]. P. K. Sah, A. M. Raj, and A. K. Sah, “Fibonacci sequence with golden ratio and its application,” Int. j. math. trends technol., vol. 66, no. 3, pp. 28–32, 2020.

[2]. V. Chetan, correlation of Fibonacci sequence and golden ratio with its application in engineering and science.

[3]. S. Sinha, “The Fibonacci Numbers and Its Amazing Applications,” *International Journal of Engineering Science Invention (IJESI)*, vol. 6, no. 9, pp. 7–14, 2017.

[4]. S. Sarma, Gauhati University, and R. K. Bhuyan, “Fibonacci number, golden ratio and their connection to different floras,” Int. j. math. trends technol., vol. 61, no. 2, pp. 95–99, 2018

[5]. A. R. Watson, “The Golden relationships: An exploration of Fibonacci Numbers and Phi,” 2017.

[6]. G. A. Hisert, “The use of cragmont Fibonacci matrices in analyzing and cataloging identities with powers of Fibonacci and Lucas numbers,” *JP J. Algebra Number Theory Appl.*, vol. 41, no. 1, pp. 95–119, 2019.

[7]. P. K. Sah, A. M. Raj, and A. K. Sah, “Fibonacci sequence with golden ratio and its application,” *Int. j. math. trends technol.*, vol. 66, no. 3, pp. 28–32, 2020.

[8]. M. Garg and A. Garg, “Advanced fibonacci sequence with golden ratio,” *International Journal of Scientific and Engineering research*, vol. 5, no. 6, pp. 288–339, 2014

[9]. B. Kaygin, B. Balçin, C. Yildiz, and S. Arslan, “The effect of teaching the subject of Fibonacci numbers and golden ratio through the history of mathematics,” Procedia Soc. Behav. Sci., vol. 15, pp. 961–965, 2011.

[10]. N. Parveen, “Fibonacci in nature,” *International Journal of Mathematical Education and Science*, vol. 15, no. 5, pp. 45–69, 2010.

[11]. “Lokenath, a short history of fibonacci and golden number with their application “International,” *International Journal of Mathematical Education in Science and Technology*, vol. 42, no. 3, pp. 337–367, 2011.

[12]. A. Stakhov, “Fundamentals of a new kind of mathematics based on the Golden Section,” Chaos Solitons Fractals, vol. 27, no. 5, pp. 1124–1146, 2006.

[13]. A. P. Stakhov, “The Generalized Principle of the Golden Section and its applications in mathematics, science, and engineering,” *Chaos Solitons Fractals*, vol. 26, no. 2, pp. 263–289, 2005.

[14]. M. Křížek, L. Somer, and A. Šolcová, “Fibonacci and Lucas numbers,” in *From Great Discoveries in Number Theory to Applications*, Cham: Springer International Publishing, 2021, pp. 151–181.